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Modelling of the flow of streams of cohesionless and cohesive bulk materials in a conveyor discharge point of a descending belt conveyor

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Abstract

The paper presents the analysis of the condition of the flow of cohesionless and cohesive bulk solids in a conveyor discharge point of a descending belt. The analysis was carried out for stationary flows at high velocities. It presents mathematical methods of description of the velocity of materials leaving a conveyor discharge point with a descending conveyor belt, as well as final equations which enable the determination of the velocity of material after it has left the discharge point, with the accuracy sufficient for practical use. Next, the obtained mathematical descriptions for the mentioned material groups have been compared. The usefulness of the equations in determining the velocity of the material beyond the point, which enable to omit the indirect equations, has been demonstrated. Finally, the difference between the values of the velocities obtained with the proposed and indirect equations as well as the relative error for the proposed method have been calculated.

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Keywords: pouring points; discharge point; bulk materials; cohesion; adhesion; modeling of the flow; material velocity

1. Introduction

Discharge points in transport conveying systems built on the basis of belt conveyors are used to move material on different levels of the system. Most of the points use the gravity for material movement, e.g. impact points or distributing points. In a discharge point, material is conveyed on a driven belt, and is separated from the belt on a conveyor pulley.

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Belt conveyors in discharge points can move material at high velocities, above 3 [m/s] or at small velocities, below the mentioned value. The analysis of material behavior in a conveyor discharge point in which material travels at a velocity below 3 [m/s] was presented in [8]. It considered cohesive and cohesionless bulk solids. However, the assumption that belt moves slowly, meant that the proposed solutions cannot be applied to fast conveyors. The methods of determining the velocity of material stream beyond the fast conveyors described in [8] included the influence of the air drag coefficient, as a factor limiting the flow, but they did not include adhesion. Equations describing the velocity beyond the point, presented in paper [11], were based only on kinematic and geometric parameters of the outflow. They included material parameters but did not include the adhesion factor. Specialized appliances used for accelerating and discharge the stream of bulk material, i.e. discharge belt conveyors, are described in [9]. However, both the geometry and the different structural solutions of discharge methods, prevented the application of the proposed in [9] solutions describing the outflow of the material from fast conveyor pulley. Simple engineering calculations describing the velocity of the material leaving a belt conveyor are described in [2]. The adhesion and forces affecting the material stream were not taken into consideration. However, the friction coefficient resulting from the friction of the material against the conveyor belt is taken into account. The adhesion effect is also omitted in [4]. Paper [7] describes the trajectory of the throw of material depending on the velocity of a conveyor belt, but it does not analyze the velocity of the material beyond the point in the inflowing places of another point. Paper [3] focuses on the review and comparison of the prediction methods of material trajectories beyond the pulley, both for slow and fast conveyors. Paper [5] proposes equations designed for prediction of the resultant velocity of the material on the final section of cooperating points. The analysis is carried out for cohesionless materials and for belt conveyors with an ascending belt. However, the analysis of the accuracy of the velocity beyond the discharge point is not taken into consideration. Paper [6] presents a solution for belt conveyors with ascending belts and for cohesive materials. The proposed solutions was analyzed in terms of their usefulness to determine the velocity changes for very small modifications of the angle of outflow trajectory.

In the hereto paper, geometric, kinematic and dynamic conditions of the flow of material in a discharge point were taken into account, as well as indirect equations, described in [12], allowing the determination of the velocity of material beyond the discharge point cooperating with an impact plate. These equations allow the determination of the angle and velocity of the material flowing into the plate.

The equations utilize the distance between the plate and the head pulley. The presented hereto study includes adhesion, which is often omitted in fast conveyors analyses.

The results obtained with indirect equations were compared with the results obtained with the use of equations proposed in this paper, for belt conveyors with descending belts. Both, cohesionless and cohesive materials were taken into account.

2. Analysis of the flow of cohesionless and cohesive bulk materials in a conveyor discharge point for a descending belt

The analysis of the flow of cohesionless and cohesive bulk materials is shown in figure 1a and 1b respectively. The analysis was carried out for stationary flow of the bulk materials, travelling at high velocities, i.e. above 3 [m/s]. Belts without cleats where taken into consideration.

Figures 1a and 1b illustrate the main quantities, which affect the process of discharging bulk materials form a head pulley of a conveyor. The analysis of the outflow includes dynamic, kinematic and geometric conditions of the elementary mass dm in a discharge point for a belt conveyor with a descending belt. The analysis assumes that the material stream is separated from pulley in point A, in which the belt overlaps the head pulley of the conveyor. For both cohesionless and cohesive bulk solids, the material may leave the belt if the following condition is met:

$$\frac{v^2}{R} > g \cdot cos\,\alpha \tag{1}$$

where: v- velocity of the belt [m/s], R-pulley radius [m], g-acceleration of gravity[m/s^2], α-angle of the conveyor inclination [^{o}].

Following assumption for cohesive materials for belt conveyors operating with high velocities is met (according to [8]):

$$\frac{F_r - A_d}{\cos\alpha} \geq 1 \tag{2}$$

and:

$$A_d = \frac{\sigma_a}{\gamma \cdot h_m} \tag{3}$$

$$F_r = \frac{v^2}{g \cdot R} \tag{4}$$

where: v-velocity of the belt [m/s], γ-specific gravity of bulk material [N/m^3], σ_a-adhesion [N/m^2], h_m-thickness of the material stream [m], g-acceleration of gravity [m/s^2], R-pulley radius [m], F_r-Froude number, A_d-adhesion number.

For cohesionless materials, equation 2 is insignificant, because adhesion phenomenon does not occur at the contact surface: conveyor belt – material.

In a discharge point transporting cohesionless materials, the material is affected by forces which describe dynamic conditions (fig. 1 a). The forces include: gravitational force dG [N], centrifugal force dF_c [N], normal force dN_{xs} [N], tangential force dT_{xs} [N], formed between the discharged material and the conveyor belt. Inertial force dJ [N] also should be taken into consideration. It is formed as a result of forces such as: gravitational force dG, centrifugal force dF_c, and surface forces dN_{xs}, dT_{xs}.

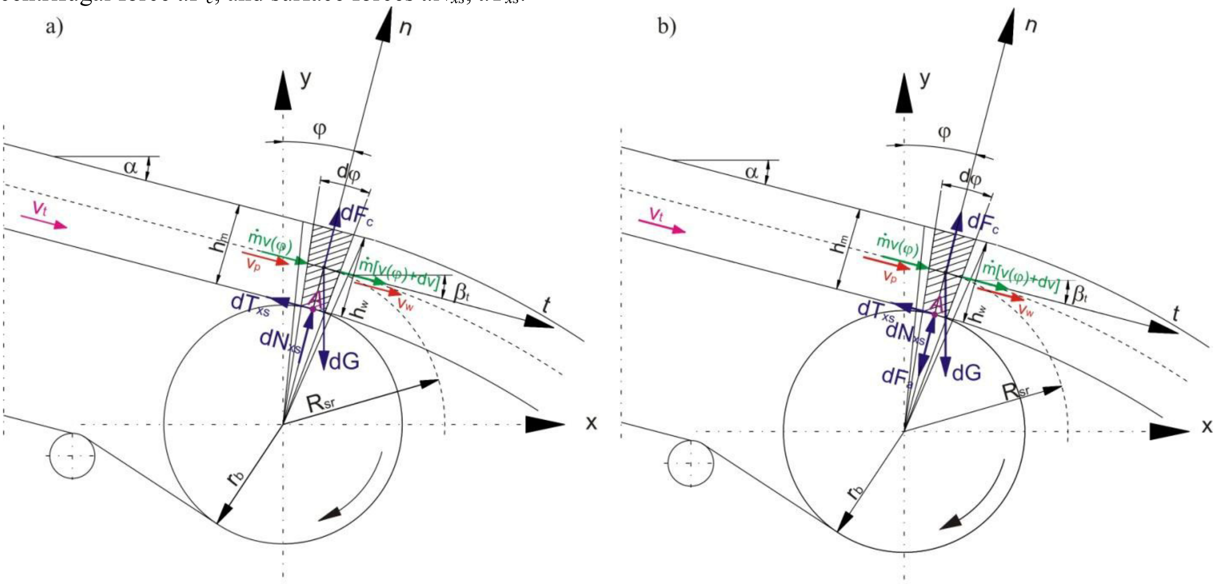

Fig. 1. Kinematic, geometric and dynamic conditions of the flow of bulk material on a head pulley of a belt conveyor with a descending belt: a) cohesionless material, b) cohesive material [own elaboration based on 8]

Kinematic conditions of the flow are described by the velocity of conveyor belt v_t [m/s], velocity of the flow of material stream into the head pulley v_p [m/s] equal to the velocity of the belt, velocity of material leaving the head pulley v_w [m/s].

The following geometric conditions of the outflow of cohesionless material from a head pulley in a discharge point with a descending belt are assumed: angle of conveyor descent α [°], angle of a material stream flowing out of a head pulley β_t [°], angle coordinate which describes the position of infinitesimal mass of material on a head pulley φ [°], radius of the average curvature of material stream R_{sr} [m], radius of a head pulley r_b [m], thickness of the layer of material stream at the moment it flows onto a head pulley h_m [m], thickness of the layer of material stream leaving a head pulley h_w [m].

The analysis carried out for cohesive materials also includes adhesion force dF_a [N]. Adhesion force acts on the contact surface, between the transported material - and the material which the belt is made of. Internal force dJ [N], for these kinds of materials, results from the same forces as for cohesionless materials, i.e. gravitational force dG, centrifugal force dF_c, surface forces dN_{xs}, dT_{xs}, and also adhesion force dF_a.

The assumptions for kinematic and geometric conditions of the outflow of cohesive materials from a head pulley (fig. 1 b) are the same as for cohesionless materials.

Thus, the adhesion force included in the analysis of cohesive materials is the only difference in the assumptions for both cases. The remaining conditions of the outflow of material presented in fig. 1a and 1b are the same.

3. Mathematical model of the flow of cohesionless and cohesive bulk material in a conveyor discharge point with a descending belt

The system of equations, which enables deduction of a relationship determining the value of the velocity v(φ) [m/s] for cohesionless and cohesive materials leaving a discharge point, is as follows:

- equation of continuity, which for cohesionless and cohesive materials takes the same form:

$$\dot{m} = \rho \cdot v(\varphi) \cdot A(\varphi) \tag{5}$$

- equation of equilibrium for cohesionless materials:

$$\dot{m}(\vec{v} + d\vec{v}) - \dot{m}\vec{v} = d\vec{G} + d\vec{F}_c + d\vec{N}_{xs} + d\vec{T}_{xs} \tag{6}$$

- equation of equilibrium for cohesive materials:

$$\dot{m}(\vec{v} + d\vec{v}) - \dot{m}\vec{v} = d\vec{G} + d\vec{F}_c + d\vec{F}_a + d\vec{N}_{xs} + d\vec{T}_{xs} \tag{7}$$

- equation describing contact friction condition on the surfaces of walls enclosing the path of the flow of a bulk material stream for static conditions and cohesionless materials (according to [8]):

$$\tau_{ws} = \sigma_n \mu_{xs} \tag{8}$$

- equation describing contact friction condition on the surfaces of walls enclosing the path of the flow of a bulk material stream for static conditions and cohesive materials (according to [8]):

$$\tau_{ws} = (\sigma_n + \sigma_a)\mu_{xs} \tag{9}$$

where μ_{xs} is contact friction coefficient for static conditions.

The general shape of the system of equations for cohesionless and cohesive materials is consists of three equations, and for cohesionless materials takes the form:

$$\begin{cases} \dot{m} = \rho \cdot v(\varphi) \cdot A(\varphi) \\ \dot{m}(\vec{v} + d\vec{v}) - \dot{m}\vec{v} = d\vec{G} + d\vec{F}_c + d\vec{N}_{xs} + d\vec{T}_{xs} \\ \tau_{ws} = \sigma_n \mu_{xs} \end{cases} \tag{10}$$

and for cohesive materials, the form of the system of equations has the following structure:

$$\begin{cases} \dot{m} = \rho \cdot v(\varphi) \cdot A(\varphi) \\ \dot{m}(\vec{v} + d\vec{v}) - \dot{m}\vec{v} = d\vec{G} + d\vec{F}_c + \vec{F}_a + \left|d\vec{N}_{xs}\right| + \left|d\vec{T}_{xs}\right| \\ \tau_{ws} = (\sigma_n + \sigma_a)\mu_{xs} \end{cases} \tag{11}$$

Projection of the equations 6 and 7 onto direction of assumed coordinate system <*n,t*> will generate systems of equations as a function of angular parameter φ. For cohesionless materials:

$$\sum{}_n = \left|d\vec{N}_{xs}\right| + \left|d\vec{F}_c\right| - \left|d\vec{G}\right| \cos\varphi = 0 \tag{12}$$

$$\sum{}_t = -\dot{m}v(\varphi) + \dot{m}[v(\varphi) + dv] - \left|d\vec{G}\right| \sin\varphi + \left|d\vec{T}_{xs}\right| = 0 \tag{13}$$

and for cohesive materials:

$$\sum{}_n = \left|d\vec{F}_c\right| - \left|d\vec{F}_a\right| + \left|d\vec{N}_{xs}\right| - \left|d\vec{G}\right| \cos\varphi = 0 \tag{14}$$

$$\sum{}_t = -\dot{m}v(\varphi) + \dot{m}[v(\varphi) + dv] + \left|d\vec{T}_{XS}\right| - \left|d\vec{G}\right| \sin\varphi = 0 \tag{15}$$

Differential equations are obtained by solving the above systems of equations. For cohesionless materials the equation takes the form:

$$\frac{dv^2(\varphi)}{d\varphi} - 2\mu_{xs}\, v^2(\varphi) = -2\, g\, R(\varphi)[\mu_{xs} \cos\varphi - \sin\varphi] \tag{16}$$

and for cohesive materials, including adhesion, it takes the following form:

$$\frac{dv^2(\varphi)}{d\varphi} - 2\, \mu_{xs} v^2(\varphi) = -2\, g\, R(\varphi)\left[\mu_{xs} \cos\varphi - \sin\varphi + \frac{2\mu_{xs}\sigma_a}{\gamma h(\varphi)}\right] \tag{17}$$

Equations 16 and 17 are the special case of the Bernoulli equation, which can be written in the form (according to [10]):

$$y' + P(\varphi)y = Q(\varphi) \tag{18}$$

where, for cohesionless materials, particular parts of the equation are as follows:

$$y = v^2(\varphi), \quad P(\varphi) = -2\mu_{xs}, \quad Q(\varphi) = -2\, g\, R(\varphi)[\mu_{xs} \cos\varphi - \sin\varphi] \tag{19}$$

and for cohesive materials:

$$y = v^2(\varphi), \quad P(\varphi) = -2\mu_{xs}, \quad Q(\varphi) = -2\,g\,R(\varphi)\left[\mu_{xs}\cos\varphi - \sin\varphi + \frac{2\mu_{xs}\sigma_a}{\gamma h(\varphi)}\right] \tag{20}$$

Parameters appearing in systems of equations 10 and 11 include: mass flow $\dot{m}$ [kg/s], material bulk density ρ [kg/m^3], cross section of the stream $A(\varphi)$ [m^2], which can be described with the same relation for both cohesionless and cohesive materials [8, 5, 6]:

$$A(\varphi) = B \cdot h(\varphi) = \frac{\dot{m}}{\rho \cdot v(\varphi)} \tag{21}$$

where B is width of the belt [m].

Gravitational force dG [N], which for both kinds of material is determined in the same way (according to [10]):

$$\left|d\vec{G}\right| = \rho\,g\,R(\varphi)\,A(\varphi)\,d\varphi = \gamma\,R(\varphi)\,A(\varphi)\,d\varphi \tag{22}$$

where specific gravity of bulk material γ [N/m^3]:

$$\gamma = \rho\,g \tag{23}$$

g is the gravitational acceleration [m/s^2], $R(\varphi)$ is an average radius of curvature of the stream [m].

Centrifugal force and force normal to the surface of the belt, in both material cases can be described in the same way [5, 6]:

$$\left|d\vec{F}_c\right| = \dot{m}\,v(\varphi)\,d\varphi \tag{24}$$

$$\left|d\vec{N}_{xs}\right| = \sigma_n\,B\,R(\varphi)\,d\varphi \tag{25}$$

σ_n is normal stress [kN/m^2].

In the case of a friction force, the equations have different forms, i.e. for cohesionless materials [6]:

$$\left|d\vec{T}_{xs}\right| = \tau_{ws}\,R(\varphi)\,B\,d\varphi = \sigma_n\,\mu_{xs}\,R(\varphi)\,B\,d\varphi = \mu_{xs}\left|d\vec{N}_{xs}\right| \tag{26}$$

.

where τ_{ws} is shear stress [kN/m^2], and for cohesive materials [5]:

$$\left|d\vec{T}_{xs}\right| = \tau_{ws}\,R(\varphi)\,B\,d\varphi = \sigma_n\,\mu_{xs}\,R(\varphi)\,B\,d\varphi + \sigma_a\mu_{xs}\,R(\varphi)\,B\,d\varphi = \mu_{xs}\left|d\vec{N}_{xs}\right| + \mu_{xs}\left|d\vec{F}_a\right| \tag{27}$$

Adhesion force occurring in equation 14 is defined by relationship:

$$\left|d\vec{F}_a\right| = \sigma_a B R(\varphi) d\varphi \tag{28}$$

Equation 18 was integrated and the resultant formulations allow the determination of the velocity of the material beyond the discharge point with a descending belt in relation to the angle of the outflow, for both cohesionless and cohesive materials.

The form of equation for cohesionless materials is as follows:

$$v_w = \sqrt{C_w\, e^{2\mu_{xs}\varphi} + \frac{2\,g\,R(\varphi)}{4\mu_{xs}^2 + 1}\left[\cos\varphi\left(2\mu_{xs}^2 - 1\right) - 3\mu_{xs}\sin\varphi\right]} \tag{29}$$

and for cohesive materials it takes the following form:

$$v_w = \sqrt{C_w\, e^{2\mu_{xs}\varphi} + \frac{2\,g\,R(\varphi)}{4\mu_{xs}^2 + 1}\left[\cos\varphi\left(2\mu_{xs}^2 - 1\right) - 3\mu_{xs}\sin\varphi\right] + \frac{2gR(\varphi)\sigma_a}{\gamma h(\varphi)}} \tag{30}$$

Integration constant C_w for the equation 29 has the form:

$$C_w = e^{-2\mu_{xs}\varphi}\left\{v_p^2 - \frac{2\,g\,R(\varphi)}{4\mu_{xs}^2 + 1}\left[\cos\varphi\left(2\mu_{xs}^2 - 1\right) - 3\mu_{xs}\sin\varphi\right]\right\} \tag{31}$$

and for equation 30:

$$C_w = e^{-2\mu_{xs}\varphi}\left\{v_p^2 - \frac{2\,g\,R(\varphi)}{4\mu_{xs}^2 + 1}\left[\cos\varphi\left(2\mu_{xs}^2 - 1\right) - 3\mu_{xs}\sin\varphi\right] - \frac{2gR(\varphi)\sigma_a}{\gamma h(\varphi)}\right\} \tag{32}$$

As can be seen, both equations differ by the element including adhesion. The analysis demonstrates that in the case of additional parameters including material properties, the relationship describing the velocity contains an additional component including analyzed parameter.

The boundary conditions that allow determining the velocity of the transported material beyond the discharge point remain the same for cohesionless and cohesive materials.

To determine the integration constant the following should be included:

$$\varphi = \alpha \tag{33}$$

$$R(\varphi) = R(\alpha) = R_0 = r_b + 0{,}5h_m \tag{34}$$

$$h_m = \frac{\dot{m}}{\rho v_t B} \tag{35}$$

$$v(\varphi) = v(\alpha) = v_0 = v_t \tag{36}$$

$$A(\varphi) = A(\alpha) = A_0 = \frac{\dot{m}}{v(\alpha)\,\rho} = \frac{\dot{m}}{v_t\,\rho} = h_m\,B \tag{37}$$

Velocity v_w can be determined by following boundary condition:

$$\varphi = \beta_t \tag{38}$$

The value of the velocity of material beyond the point is obtained by applying the simple iteration method, taking initial conditions determined from relations 33-38.

To obtain the exact approximation of the velocity v_w of material leaving the point, the following relation should be satisfied:

$$\left|\frac{v_{wn} - v_{w(n-1)}}{v_{wn}}\right| \leq \delta_{vw} \ [\%] \tag{39}$$

where δ_{vw} is an acceptable relative deviation of the estimated velocity v_w. It can take the value 1%-2%.

Parameter β_t varies depending on the distance between the head pulley and the point beyond it where the velocity of the material is analyzed. The proposed equations were verified in terms if they conform the assumption that the velocity of the material measured directly beyond the pulley is equal to the velocity of the belt, and they were found to have followed the assumptions. Additionally, the proposed equations can also be used for determining the velocity beyond the discharge point, depending on the angle of the material flowing into another point. Then, angle β_t is equal to the angle at which the material is dumped onto another surface (e.g. impact plate). This approach represents a major simplification, however allows achieving results which are sufficient for practical use. An example of the proposed equations is presented in the further part of this paper.

4. Determination of the velocity of the material at an arbitrary distance from the discharge point

Table 1 presents an example of the application of the proposed solutions. The example demonstrates the usefulness of the proposed equations for the determination of the velocity of material beyond the discharge point irrespectively to the distance in which it is discharged, taking into account the angle at which it flows into another pouring point. It includes equations for cohesionless and cohesive materials. Apart from the proposed equations, Table 1 also presents the results obtained by indirect equations, as well as an estimated percentage error in relation to the indirect equations. It also shows the differences between the obtained results. The conditions of discharging the material from the pulley were assumed for the indirect equations.

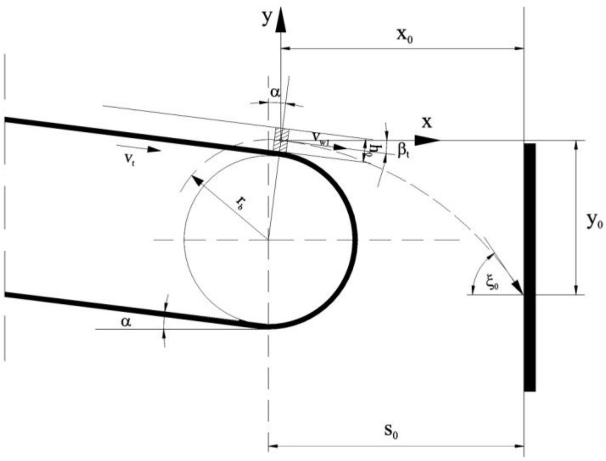

Fig 2. Indirect parameters which allow determining the velocity of the material flowing into another discharge point (impact plate) – pictorial figure [elaboration based on 10]

Four indirect equations allow determining the velocity beyond the discharge point described in [12] include (fig. 2):

- an angle of material flowing into another point ξ_0:

$$\xi_0 = \left|\operatorname{arctg}\left(\operatorname{tg}\alpha - \frac{g \cdot x_0}{2 \cdot v_{w1}^2 \cdot \cos^2 \alpha}\right)\right| \tag{40}$$

- the velocity of material flowing into another point v_0:

$$v_0 = v_{w1} \cdot \cos \beta_t \cdot \sqrt{tg^2 \xi_0 + 1} \tag{41}$$

- the distance parameters according to fig. 2:

$$x_0 = s_0 + \left(r_b + 0{,}5 \cdot h_0\right) \cdot \sin \beta_t \tag{42}$$

$$y_0 = tg\alpha \cdot x_0 - \frac{g \cdot x_0^2}{2 \cdot v_{w1} \cdot \cos^2 \alpha} \tag{43}$$

where (fig. 2) x_0-distance between material on the head pulley and the impact plate [m], y_0- distance between material on the head pulley and the impact point [m].

Table 1. Determination of the velocity of material beyond the discharge point depend on the analyzed angle at which material flows into another point [own elaboration]

Cohesionless materials	Cohesive materials
• required mass flow $\dot{m}$ = 350 [kg/s], • belt velocity v_p = 4,5 [m/s] • belt width B_t = 0,8 [m], • pulley radius r_b = 0,25 [m], • angle of inclination of belt conveyor α = 15 [°], • gravitational acceleration g = 9,81 [m/s²], • material bulk density ρ = 850 [kg/m³], • specific gravity γ = 8340 [N/m³], • friction coefficient for static conditions μ_{xs} = 0,50, • angle at which bulk material flows into another point $\beta_t = \xi_0$ = 19 [°] (assumption for proposed equations), • distance between the head pulley and the point at which material is discharged s_0= 1,1 [m]	• required mass flow $\dot{m}$ = 350 [kg/s], • belt velocity v_p = 4,5 [m/s] • belt width B_t = 0,8 [m], • pulley radius r_b = 0,25[m], • angle of inclination of belt conveyor α = 15 [°], • gravitational acceleration g = 9,81 [m/s²], • material bulk density ρ = 1380 [kg/m³], • adhesion σ_a = 350 [N/m²], • specific gravity γ = 13540 [N/m³], • friction coefficient for static conditions μ_{xs} = 0,50, • angle at which bulk material flows into another point $\beta_t = \xi_0$ = 19 [°] (assumption for proposed equations), • distance between the head pulley and the point at which material is discharged s_0= 1,1 [m]
Calculations • the first step of approximation boundary conditions determining integration constant $\varphi = \alpha = 15$ $v(\varphi) = v(\alpha) = v_0 = v_t = 4{,}5$ $h_m = \frac{A_{w1}}{B} = \frac{\dot{m}}{\rho v_t B} = \frac{350}{850 \cdot 4{,}5 \cdot 0{,}8} = 0{,}114$ $A_0 = A_{w1} = \frac{\dot{m}}{v_t \rho} = h_m B = 0{,}114 * 0{,}8 = 0{,}091$ $R_{w1} = R_{sr1} = r_b + 0{,}5h_m = 0{,}25 + 0{,}5 \cdot 0{,}114 = 0{,}307$ integration constant $C_{w1} = e^{-2\mu_{xs}\alpha}\left\{v_t^2 - \frac{2gR_{sr1}}{4\mu_{xs}^2+1}\left[\cos\alpha\left(2\mu_{xs}^2-1\right) - 3\mu_{xs}\sin\alpha\right]\right\}$ $C_{w1} = 11{,}860$ boundary condition determining the velocity $\varphi = \beta_t = 19$ velocity	Calculations • the first step of approximation boundary conditions determining integration constant $\varphi = \alpha = 15$ $v(\varphi) = v(\alpha) = v_0 = v_t = 4{,}5$ $h_m = \frac{A_{w1}}{B} = \frac{\dot{m}}{\rho v_t B} = \frac{350}{1380 \cdot 4{,}5 \cdot 0{,}8} = 0{,}070$ $A_0 = A_{w1} = \frac{\dot{m}}{v_t \rho} = h_m B = 0{,}070 \cdot 0{,}8 = 0{,}056$ $R_{w1} = R_{sr1} = r_b + 0{,}5h_m = 0{,}25 + 0{,}5 \cdot 0{,}070 = 0{,}285$ integration constant $C_{w1} = e^{-2\mu_{xs}\alpha}\left\{v_p^2 - \frac{2gR_{sr1}}{4\mu_{xs}^2+1}\left[\cos\alpha\left(2\mu_{xs}^2-1\right) - 3\mu_{xs}\sin\alpha\right] - \frac{2gR_{sr1}\sigma_a}{\gamma h_m}\right\}$ $C_{w1} = 11{,}261$ boundary condition determining the velocity $\varphi = \beta_t = 19$ velocity $v_{w1} = \sqrt{C_{w1} e^{2\mu_{xs}\beta_t} + \frac{2gR_{sr1}}{4\mu_{xs}^2+1}\left[\cos\beta_t\left(2\mu_{xs}^2-1\right) - 3\mu_{xs}\sin\beta_t\right] + \frac{2gR_{sr1}\sigma_a}{\gamma h_m}}$

Cohesionless materials	Cohesive materials
$v_{w1} = \sqrt{C_{w1} e^{2\mu_{xs}\beta_t} + \frac{2gR_{sr1}}{4\mu_{xs}^2+1}\left[\cos\beta_t\left(2\mu_{xs}^2-1\right)-3\mu_{xs}\sin\beta_t\right]}$ $v_{w1} = 4{,}749$ deviation value $\left\lvert\frac{v_{wn}-v_{w(n-1)}}{v_{wn}}\right\rvert\cdot 100 \le \delta_{v_w}$ $\delta_{v_{w1}} = 6{,}136 > 1\%$ approximation is insufficient • the second step of approximation boundary conditions determining integration constant $h_{w1} = \frac{A_{w2}}{B} = \frac{\dot{m}}{\rho v_{w1} B} = \frac{350}{850\cdot 4{,}749\cdot 0{,}8}0{,}107$ $A_{w2} = \frac{\dot{m}}{v_{w1}\rho} = h_{w1}B = 0{,}107\cdot 0{,}8 = 0{,}856$ $R_{w2} = R_{sr2} = r_b + 0{,}5h_{w1} = 0{,}25 + 0{,}5\cdot 0{,}107 = 0{,}303$ integration constant $C_{w2} = e^{-2\mu_{xs}\alpha}\left\{v_t^2 - \frac{2gR_{sr2}}{4\mu_{xs}^2+1}\left[\cos\alpha\left(2\mu_{xs}^2-1\right)-3\mu_{xs}\sin\alpha\right]\right\}$ $C_{w2} = 11{,}862$ boundary condition determining the velocity $\varphi = \beta_t = 19$ velocity $v_{w2} = \sqrt{C_{w2} e^{2\mu_{xs}\beta_t} + \frac{2gR_{sr2}}{4\mu_{xs}^2+1}\left[\cos\beta_t\left(2\mu_{xs}^2-1\right)-3\mu_{xs}\sin\beta_t\right]}$ $v_{w2} = 4{,}794$ deviation value $\delta_{v_{w2}} = 0{,}007 < 1\%$ • indirect equations $\alpha = \beta_t = 15$ (according to [12]) $x_0 = s_0 + \left(r_b + 0{,}5\cdot h_0\right)\cdot\sin\beta_t$ $x_0 = 1{,}179$ $\xi_0 = \left\lvert\operatorname{arctg}\left(\operatorname{tg}\beta_t - \frac{g\cdot x_0}{2\cdot v_{w1}^2\cdot\cos^2\beta_t}\right)\right\rvert$ $\xi_0 = 19$ $v_0 = v_t\cdot\cos\beta_t\cdot\sqrt{\operatorname{tg}^2\xi_0 + 1}$ $v_0 = 4{,}597$ • relative error of the determination of the velocity with both methods $\delta_{vr} = 4\%$	$v_{w1} = 4{,}780$ deviation value $\left\lvert\frac{v_{wn}-v_{w(n-1)}}{v_{wn}}\right\rvert\cdot 100 \le \delta_{v_w}$ $\delta_{v_{w1}} = 5{,}864 > 1\%$ approximation is insufficient • the second step of approximation boundary conditions determining integration constant $h_{w1} = \frac{A_{w2}}{B} = \frac{\dot{m}}{\rho v_{w1} B} = \frac{350}{1380\cdot 4{,}780\cdot 0{,}8}0{,}066$ $A_{w2} = \frac{\dot{m}}{v_{w1}\rho} = h_{w1}B = 0{,}066\cdot 0{,}8 = 0{,}052$ $R_{w2} = R_{sr2} = r_b + 0{,}5h_{w1} = 0{,}25 + 0{,}5\cdot 0{,}066 = 0{,}283$ integration constant $C_{w2} = e^{-2\mu_{xs}\alpha}\left\{v_p^2 - \frac{2gR_{sr2}}{4\mu_{xs}^2+1}\left[\cos\alpha\left(2\mu_{xs}^2-1\right)-3\mu_{xs}\sin\alpha\right] - \frac{2gR_{sr2}\sigma_a}{\gamma h_{w1}}\right\}$ $C_{w2} = 11{,}229$ boundary condition determining the velocity $\varphi = \beta_t = 19$ velocity $v_{w2} = \sqrt{C_{w2} e^{2\mu_{xs}\beta_t} + \frac{2gR_{sr2}}{4\mu_{xs}^2+1}\left[\cos\beta_t\left(2\mu_{xs}^2-1\right)-3\mu_{xs}\sin\beta_t\right] + \frac{2gR_{sr2}\sigma_a}{\gamma h_{w1}}}$ $v_{w2} = 4{,}779$ deviation value $\delta_{v_{w2}} = 0{,}013 < 1\%$ • indirect equations $\alpha = \beta_t = 15$ (according to [12]) $x_0 = s_0 + \left(r_b + 0{,}5\cdot h_0\right)\cdot\sin\beta_t$ $x_0 = 1{,}173$ $\xi_0 = \left\lvert\operatorname{arctg}\left(\operatorname{tg}\beta_t - \frac{g\cdot x_0}{2\cdot v_{w1}^2\cdot\cos^2\beta_t}\right)\right\rvert$ $\xi_0 = 19$ $v_0 = v_t\cdot\cos\beta_t\cdot\sqrt{\operatorname{tg}^2\xi_0 + 1}$ $v_0 = 4{,}593$ • relative error of the determination of the velocity with both methods $\delta_{vr} = 4\%$

Table 1 presents the analysis for both kinds of materials but for the same geometrical conditions. The relative error δ_{vr} for results achieved with proposed equations and with indirect equations amounts 4%, which is acceptable for practical conditions. The difference between the velocity calculated with the indirect equations and the velocity calculated with the proposed relations for cohesionless materials amounts 0,197 [m/s], and for cohesive materials amounts 0,186 [m/s].

It appears, that the proposed equations can be used for prediction of the velocity beyond the discharge point. The simplified equations take into account geometric and kinematic conditions, whereas the proposed equations also include dynamic parameters and others such as adhesion, friction coefficient between material and belt surface, as well as material parameters such as bulk density and specific gravity, which are not included in the simplified method.

5. Conclusions

The presented analysis of the flow conditions of material in the discharge point with descending belt and beyond it, as well as proposed equations are suitable for engineering calculations and for estimation of the flow parameters of bulk materials in place, where values of the velocity are not measured (beyond the point).

The value of the velocity is very important in terms of proper operation of the transport system. Incorrectly selected velocity can result in an overloaded discharge point and in consequence its damage. The correctly estimated velocity is also essential for maintaining the continuous efficiency of the transport system.

The proposed equations, which take into account material parameters, enable the extensive analysis of the discharge of material in a conveyor discharge point, as well as its behavior beyond the point.

Despite the fact that the analysis includes the angle of the material stream beyond the point is considered (regardless to the discharge distance), the obtained numerical values of the velocity give good results comparing to the applied equations 40-43.The analyzes of the application of the equations reveal that the smaller difference between the angle of the stream flowing out of the point and the angle of the stream flowing into a subsequent point, the smaller percentage difference in the obtained results. Another conclusion is that, the smaller distance between the head pulley and the impact point, the smaller relative error δ_{vr} is received.

The maximum error between the proposed equations and the indirect equations used for analyzing various values of the angle of material was larger than 10% (e.g. for s_0=2[m], δ_{vr}=15%). The obtained minimum percentage value for reasonable minimum distance s_0 was 2%. It reveals, that the proposed equations can be used to calculate the velocity for extensive distance.

The presented analysis did not aim at determining the trajectory of discharged material, but at providing engineers with a useful tool to predict the value of the velocity beyond the discharge point.

The equations may also be useful for the determination of the velocity for very fast conveyors, where the change of the angle of the stream flowing out of a discharge point into a subsequent point is minor (for example in impact plates).

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